BREAKOFF OF VAPOR BUBBLES FROM A HEATING SURFACE

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The breakoff diameter of bubbles on a heating surface is determined in connection with the pool boiling and forced-circulation boiling of liquids.

An important problem in the investigation of the mechanism of liquid boiling is the determination of the vapor bubble breakoff diameters on a heating surface. Despite the many articles published on this problem, the most timely of which are [1-16], so far there has been a lack of theoretical results that are in satisfactory agreement with the experimental data obtained by various authors under widely dissimilar conditions. Detailed critical surveys of the problem may be found in [17, 19].

In the present article we give the results of a determination of the breakoff diameter of vapor bubbles in boiling under the conditions of free convection as well as forced circulation; our results concur satisfactorily with the data of fifteen papers for three different liquids.

Let us consider the main forces acting on a vapor bubble situated on a heat-transmitting surface in a liquid flow (Fig. 1):

The buoyancy is described as follows:

$$P_g = \frac{4}{3}\zeta_g \pi g(\rho' - \rho'') R_d^3. \tag{1}$$

The coefficient ζ_g accounts for the difference between the volume of a spherical bubble of radius R_d and the volume of a nonspherical bubble having the same average radius; according to [9], we can set $\zeta_g = 1$.

The surface tension is determined from a relation similar to the one given in [9]:

$$P_\sigma = 2\zeta_\sigma \pi\sigma \sin\theta R_d. \tag{2}$$

The coefficient ζ_σ is to be determined later.

The drag forces are of two types: the drag P_R due to growth of the bubble, i.e., the resistive force determined by a velocity proportional to $dR/d\tau$, and the drag P_W due to forced circulation:

$$P_R = \frac{1}{2}\zeta_R \pi R_d^2 \rho' w_R^2, \tag{3}$$

$$P_w = \frac{1}{2}\zeta_w \chi_\zeta \pi \varphi_w \rho' w_{cp}^2 R_d^2, \tag{4}$$

where

$$\varphi_w = 1 - \frac{\theta}{\pi} + \frac{\sin 2\theta}{2\pi}.$$

We determine the drag coefficients ζ_R and ζ_W from the values of the Reynolds numbers (see, e.g., [20])

$$\mathrm{Re}_R = \frac{2R_d w_R}{\nu'};\ \mathrm{Re}_w = \frac{2R_d w_{av}}{\nu'}, \tag{5}$$

on the assumption that the bubble is a sphere.

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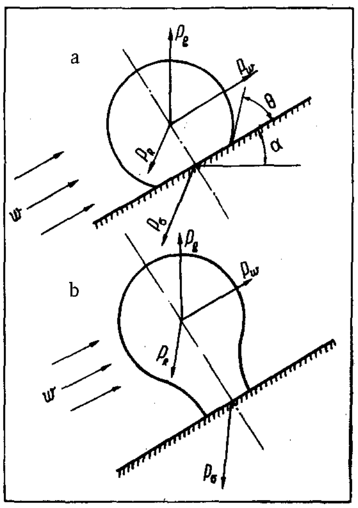

Fig. 1. Schematic of forces acting on a bubble during breakoff at constant (a) and variable (b) contact angle.

We calculate P_w and Re_w on the basis that the bubble diameter is comparable with the thickness of the boundary layer and that w varies over the height [9]. By analogy with [9], we assume that the drag for a variable speed is equivalent to that for some average speed w_{av}:

$$w_{av} = \frac{1}{(1+\cos\theta) R_d} \int_0^{(1+\cos\theta) R_d} w(y)\, dy = \varepsilon \Phi (R_d) \tilde{w}, \tag{6}$$

where for laminar flow in a circular pipe

$$\varepsilon = 2(1+\cos\theta)\left[1 - \frac{1}{3}(1+\cos\theta)\frac{R_d}{R_{fr}}\right];$$

for turbulent flow characterized by power-law velocities

$$\varepsilon = \left(1 + \frac{1}{2} n\right)(1+\cos\theta)^n; \quad \Phi = \left(\frac{R_d}{R_{fr}}\right)^n.$$

The velocity w_R can be related to the bubble growth rate $dR/d\tau|_{\tau=\tau_d} = \dot{R}_d$ by simple geometrical considerations:

$$w_R = (1+\cos\theta)\dot{R}_d. \tag{7}$$

The quantity $\dot{R}_d$ is equal to [21]

$$\dot{R}_d = \frac{2f_\theta}{(2+f_\rho)}\left[\frac{1}{\sqrt{\pi}} \mathrm{Ja} \sqrt{\frac{a'}{\tau_d}} + f_q \frac{q}{r\rho''}\right], \tag{8}$$

where

$$\mathrm{Ja} = \frac{c'\rho'\Delta T}{r\rho''}; \quad f_\rho = 1 - \frac{\rho''}{\rho'};$$

$$f_\theta = \frac{2(1+\cos\theta)}{2+\cos\theta(2+\sin^2\theta)}; \quad f_q = \frac{1}{2}(1-\cos\theta).$$

To determine the bubble growth time τ_d up to the instant of break off (departure) we use the relation [21]

$$R_d = \frac{2}{(2+f_\rho)} f_\theta \left(\frac{2}{\sqrt{\pi}} \mathrm{Ja}\sqrt{a'\tau_d} + f_q \frac{q}{r\rho''}\tau_d\right). \tag{9}$$

From this equation we readily deduce the following expression for $\dot{R}_d$ by suitable transformations:

$$\dot{R}_d = \frac{2f_\theta f_q}{(2+f_\rho)} \mathrm{Ja}\, w_q \left[1 - \left(1 + \varphi_1 \frac{\bar{R}_d}{\mathrm{Ja}} \sqrt{\frac{N_q}{\vartheta}}\right)^{-0,5}\right]^{-1},$$

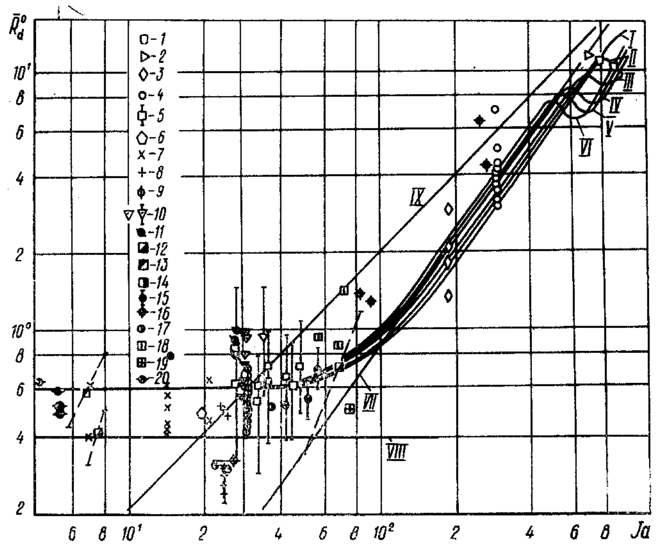

Fig. 2. Comparison of theoretical and experimental values of the dimensionless breakoff diameter for water. I-VI) Calculated according to Eqs. (18)-(21) at temperature of 40, 60, 80, 100, 120, and 140°C, respectively; VII-IX) calculated according to equations in [12, 10, 11], respectively; 1-4) experimental points from [22] at t_s = 38, 43, 65, and 52°C; 5) experimental points from [16]; 6) from [14]; 7) [25]; 8) [23]; 9) [26]; 10) [24]; 11) [10]; 12-14) [8] at p_s = 1.20 and 11 atm; 15) [18]; 16) [5]; 17) [7]; 18) [4]; 19) [28]; 20) [29] (the parameters not otherwise indicated refer to 100°C).

where

$$\overline{R}_d = \frac{1}{\sqrt{\mathrm{We}}} = \frac{R_d}{\sqrt{\dfrac{\sigma}{g(\rho'-\rho'')}}}; \quad N_q = \frac{\rho' w_q^2}{\sqrt{\sigma g(\rho'-\rho'')}}; \tag{10}$$

$$\vartheta = \frac{\rho' a'^2 \sqrt{g(\rho'-\rho'')}}{\sigma^{3/2}}.$$

These relations enable us to determine the forces P_w and P_R:

$$P_w = \frac{\pi}{2}\zeta_w \sigma R_d \chi_\zeta N_w \varepsilon^2 \overline{R}_d \varphi_w \Phi^2, \tag{11}$$

$$P_R = 2\pi\varphi_2\zeta_R\sigma R_d\,\mathrm{Ja}^2 N_q \overline{R}_d \sin\theta\left[1-\left(1+\varphi_1\frac{\overline{R}_d}{\mathrm{Ja}}\sqrt{\frac{N_q}{\vartheta}}\right)^{-0,5}\right]^{-2}. \tag{12}$$

Here

$$N_w = \frac{\rho' w^2}{\sqrt{\sigma g(\rho'-\rho'')}}; \quad \varphi_1 = \frac{\pi\left(1+\dfrac{1}{2}f_\rho\right)(1-\cos\theta)}{2f_\theta};$$

$$\varphi_2 = \frac{(1+\cos\theta)^2(f_\theta - f_q)^2}{\zeta_\sigma \sin\theta\,(2+f_\rho)}.$$

Note that a definite relationship exists between the numbers N_q and N_w and the Froude number Fr:

$$N_{w,q} = f_\rho \overline{R}_d \mathrm{Fr}_{w,q}.$$

We now set down the equilibrium conditions for the forces acting on the vapor bubble at breakoff:

$$(P_g \cos\alpha)^2 + (P_w + P_g \sin\alpha)^2 = (P_R + P_\sigma)^2. \tag{13}$$

Substituting the relations derived above into (13) and effecting suitable transformations, we obtain the following equations for $\bar{R}_d$:

$$\bar{R}_d^4 + \frac{3}{4}\sin\alpha\zeta_w\varphi_w\chi_\zeta\varepsilon^2\Phi^2 N_w\bar{R}_d^3 + \left(\frac{3}{8}\chi_\zeta\zeta_w\varphi_w\varepsilon^2\right)^2$$

$$\times N_w^2\Phi^4 R_d^2 = \left[\frac{3}{2}\zeta_\sigma\sin\theta\left(1 + \frac{\varphi_2\zeta_R \,\mathrm{Ja}^2 N_q \bar{R}_d}{\left[1 - \left(1+\varphi_1\frac{R_d}{\mathrm{Ja}}\sqrt{\frac{N_q}{\vartheta}}\right)^{-0.5}\right]^2}\right)\right]^2. \tag{14}$$

This ponderous equation cannot be solved analytically. Therefore, in order to simplify it we rely on the fact that the term containing q in (9) only slightly affects the value of R (see [21]). Then $\bar{R}_d$ can be found by solving the simpler equation

$$\bar{R}_d^6 + \frac{3}{4}\zeta_w\sin\alpha\chi_\zeta\varphi_w\varepsilon^2 N_w\Phi^2\bar{R}_d^5 + \left(\frac{3}{8}\chi_\zeta\varphi_w\zeta_w\varepsilon^2\right)^2 N_w^2\Phi^4\bar{R}_d^4$$

$$\cdot\left(\frac{3}{2}\zeta_\sigma\sin\theta\right)^2\bar{R}_d^2 - \frac{8}{9\pi^2}\zeta_R\zeta_\sigma\sin\theta(1+\cos\theta)^2\vartheta\,\mathrm{Ja}^4\bar{R}_d - \left[\frac{8}{27\pi^2}\zeta_R(1+\cos\theta)\vartheta\right]^2\mathrm{Ja}^8 = 0, \tag{15}$$

in which

$$\mathrm{Re}_R = \frac{16(1+\cos\theta)}{9\pi}\cdot\frac{\mathrm{Ja}^2}{\mathrm{Pr}'}.$$

To test the reliability of the results we consider some special cases.

In the case of pool boiling on a horizontal surface ($\alpha = 0$, $w = 0$) we obtain a third-degree equation from (15):

$$\bar{R}_d^3 - \frac{3}{2}\zeta_\sigma\sin\theta\bar{R}_d - \frac{8(1+\cos\theta)^2}{27\pi^2}\zeta_R\vartheta\,\mathrm{Ja}^4 = 0. \tag{16}$$

The form of the solution is determined by the sign of the quantity Q:

$$Q = [\psi\zeta_R\vartheta\,\mathrm{Ja}^4]^2 - \frac{1}{8}\zeta_\sigma^3\sin^3\theta, \tag{17}$$

where

$$\psi = \left[\frac{2(1+\cos\theta)}{3\sqrt{3}\pi}\right]^2.$$

If

$$\mathrm{Ja}_{\lim} \geqslant 2.21\frac{(\zeta_\sigma\sin\theta)^{3/8}}{(1+\cos\theta)^{1/2}(\zeta_R\vartheta)^{1/4}},$$

then $Q > 0$, and the solution of Eq. (16) has the form

$$\bar{R}_d = \left(\psi\zeta_R\vartheta\,\mathrm{Ja}^4 + \sqrt{Q}\right)^{\frac{1}{3}} + \left(\psi\zeta_R\vartheta\,\mathrm{Ja}^4 + \sqrt{Q}\right)^{\frac{1}{3}}. \tag{18}$$

For large numbers $\mathrm{Ja} > 100$ the second term of Eq. (17) is negligible, whereupon we obtain the following simpler relation for the breakoff radius:

$$\bar{R}_d = \frac{2}{3\sqrt[3]{\pi^2}}(1+\cos\theta)^{2/3}(\zeta_R\vartheta)^{1/3}\mathrm{Ja}^{4/3}. \tag{19}$$

If $Q < 0$, the solution (16) is determined in general from the relation

$$\bar{R}_d = \sqrt{2\zeta_\sigma\sin\theta}\cos\left[\frac{1}{3}\arccos\frac{\sqrt{2}\psi\zeta_R\vartheta\,\mathrm{Ja}^4}{(\zeta_\sigma\sin\theta)^{3/2}}\right]. \tag{20}$$

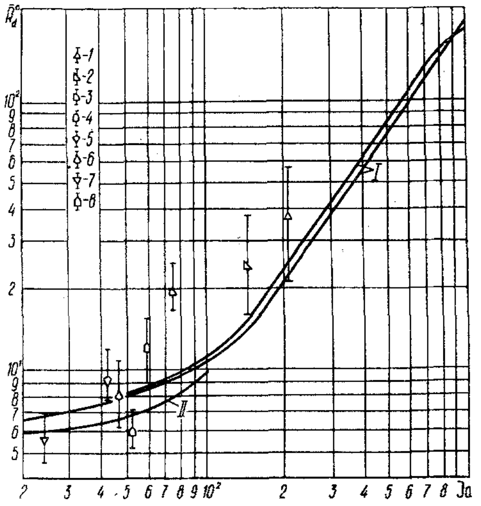

Fig. 3

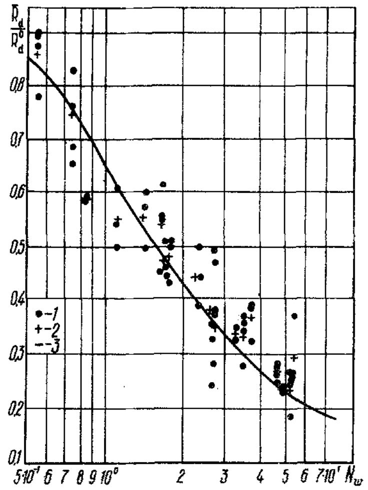

Fig. 4

Fig. 3. Comparison of theoretical and experimental values of the dimensionless breakoff diameter for methanol and n-pentane. I) Calculated according to (18)-(21) for methanol at t_s = 25 to 65°C; II) calculated according to (18)-(21) for n-pentane; 1-6) experimental data of [22] for methanol at p_s = 134, 204, 304, 397, 540, and 760 mm Hg; 7-8) experimental data of [22] for n-pentane at p_s = 760 and 524 mm Hg, respectively.

Fig. 4. Relative breakoff radius $\overline{R}_d/\overline{R}_d^0$ versus number N_w for water. 1) Experimental points; 2) average values; 3) calculated according to (23).

For small Jakob numbers (Ja < 20) the solution has the form

$$\overline{R}_d = \sqrt{\frac{3}{2}\,\zeta_\sigma \sin\theta}. \tag{21}$$

An analysis of Eqs. (18)-(21) leads to the following conclusions. For large temperature differentials (large Ja) the bubble cannot form a "stem" and departs essentially at a constant contact angle (see Fig. 1a), corresponding to $\zeta_\sigma = 1$ (T mode). In this case Eq. (19), apart from a constant multiplier, goes over to Ruckenstein's equation [12]. For small temperature differentials (small Ja) surface tension forces play a decisive role. In this case bubble breakoff occurs from an elongated "stem" formation (see Fig. 1b) (L mode), and Eq. (21) corresponds to Fritz's equation [10]; according to [9], $\zeta_\sigma = 0.36$.* Between these two modes, in the interval of limiting Jakob numbers Ja_{lim}, is a certain transition mode.

In order to test the foregoing results we borrowed the experimental results of fifteen papers, in which such liquids as water, methanol, and n-pentane were investigated. The theoretical and experimental values obtained for water are shown in Fig. 2, along with the theoretical curves calculated according to the equations of Ruckenstein (curve VII) and Fritz (VIII) and according to Eqs. (18)-(21) at various temperatures (I through VI). The equations proposed above yield better quantitative as well as qualitative agreement with the experimental results. The experimental and theoretical values for methanol and n-pentane are given in Fig. 3.

We now consider forced-circulation boiling of a liquid. In this case it is impossible to solve Eq. (15) for $\overline{R}_d$. We can only determine $\overline{R}_d$ in implicit form:

$$N_w = \frac{1}{\chi_\zeta \zeta_w \varphi_w \varepsilon^2}\left\{\left[\frac{8}{3}\sin\alpha\,\frac{\overline{R}_d}{\Phi^2} - B(\overline{R}_d,\ \Phi,\ \psi,\ \theta)\right]^{\frac{1}{2}} - \frac{8}{3}\sin\alpha\,\frac{\overline{R}_d}{\Phi^2}\right\}. \tag{22}$$

For boiling in a horizontal pipe this relation is simplified:

$$N_w = \frac{8}{3\overline{R}_d^2\varepsilon^2\Phi^2\chi_\zeta\zeta_w\varphi_w}\left[\left(\frac{3}{2}\,\zeta_\sigma\sin\theta\,\overline{R}_d + \frac{\varphi'}{3}\,K_d^3\right)^2 - \overline{R}_d^6\right]^{0,5}, \tag{23}$$

*It is interesting to note that a comparison of Eq. (21) with the corresponding equation in [3] yields $\zeta_\sigma = (\sin\theta)^{-0.5}$.

where

$$\varphi' = \frac{8}{9\pi^2}(1+\cos\theta)^2;\ K_d = \sqrt[3]{\zeta_R \vartheta\,\mathrm{Ja}^4}.$$

For small values of $\bar{R}_d$ we can derive an approximate relation for the direct dependence of $\bar{R}_d$ on N_w. In this case the last two terms under the radical in Eq. (23) can be dropped, yielding a cubic equation in $\bar{R}_d$:

$$\bar{R}_d^3 - \left(\frac{4\zeta_\sigma \sin\theta}{\chi_\xi \zeta_w \varphi_w \varepsilon^2 \Phi^2 N_w}\right)^2 \bar{R}_d - \frac{64}{9}\cdot\frac{\varphi'\zeta_\sigma \sin\theta K_d^3}{(\chi_\xi \zeta_w \varphi_w \varepsilon^2 \Phi^4 N_w)^2} = 0,$$

the solution of which is also determined by the sign of the discriminant of the equation.

To test the results we borrowed the experimental data of [9]. The calculated and experimental values are compared in Fig. 4, which gives the ratio of the breakoff radius $\bar{R}_d$ to the value of the radius $\bar{R}_d^0$ for pool boiling as a function of the number N_w.

NOTATION

R_d	is the breakoff radius;
ρ', ρ''	are the densities of liquid and vapor;
P	is the force;
σ	is the surface tension;
θ	is the contact angle;
w	is the velocity;
ν	is the kinematic viscosity;
d_e	is the equivalent diameter;
q	is the heat flux;
a	is the thermal diffusivity;
α	is the angle of surface relative to horizontal;
ξ	is the force coefficient;
w_{av}	is the integral-average velocity over bubble surface;
$\tilde{w}$	is the velocity in flow;
χ_ξ	is the coefficient accounting for the deviation of the drag obtained at the integral-average velocity w_{av} (or $Re_{w_{av}}$) from the average drag (χ_ξ is assumed to be equal to 9 according to [9]).

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